$$T(n) = 2T\left(\frac{n}{2}\right) + c_1 n + c_2 n$$

$$= 2T\left(\frac{n}{2}\right) + cn \qquad c = c_1 + c_2$$

$$T(n) = 2T\left(\frac{n}{2}\right) + \Theta\left(\overset{\frac{n}{2}}{\cancel{n}}\right) + \Theta(n)$$

$$= 2T\left(\frac{n}{2}\right) + \Theta(n)$$

$$\sum_{i=1}^{n} \Theta(n) = \sum_{i=1}^{n} c \cdot n = \overbrace{cn + cn + \cdots + cn}^{n}$$

$$= cn^2 = \Theta(n^2)$$

We tend not to write $\Theta\left(\frac{n}{2}\right)$ since it's the same as $\Theta(n)$

Let's compare growth rates of some common functions

$\frac{3}{2}n^2 + 7n - 4$ vs $8n^2$ $\Theta(n^2)$

$$\lim_{n\to\infty} \frac{3/2\, n^2 + 7n - 4}{8n^2} = \lim_{n\to\infty} \frac{3n+7}{16n} = \lim_{n\to\infty} \frac{3}{16}$$

L'Hopitals Rule

$$= \frac{3}{16}$$

$$\frac{3}{2}n^2 / 8n^2 = \frac{3}{16}$$

$\log_2 n$ vs $\sqrt{n}$

$$\lim_{n\to\infty} \frac{\log_2 n}{\sqrt{n}} = \lim_{n\to\infty} \frac{\frac{1}{n}\cdot\frac{1}{\ln 2}}{\frac{1}{2}n^{-1/2}}$$

$$= \lim_{n\to\infty} \frac{2\sqrt{n}}{(\ln 2)\, n} = \lim_{n\to\infty} \frac{2}{\ln 2\cdot\sqrt{n}}$$

$$= 0$$

$\log_2 n$ is asymptotically slower growing than $\sqrt{n}$

$\log_2 n <_{\lim} \sqrt{n}$

$$\log_a n = \frac{\log_b n}{\log_b a}$$

constant

any constant

$\log_a n = \Theta(\log_b n)$

$\log_b(n) = \Theta(\log_a n)$

constant

$\ln$ $\quad \log_e n$

$\log_2 n$

$\log_{10} n$

$$\log_2 n = \frac{\ln n}{\log_e 2}$$

Common to not show any base for log when using asymptotic notation since it has no meaning

$$T(n) = 2T\left(\frac{n}{2}\right) + \Theta(n) = \Theta(n \log n)$$

Definition of O

$f(n) = O(g(n))$ if $\exists$ constants c, n_0

$\forall n \geq n_0 \quad f(n) \leq c \cdot g(n)$

I'll refer to this as the constant hidden in the asymptotic notation

Suppose we designed 5 algs for closest pair alg

~~Alg 1~~ $\cancel{\Theta(n^2)}$

Alg 2 $\Theta(n \log n)$

~~Alg 3~~ $\cancel{\Omega(n^2)}$

~~Alg 4~~ $\cancel{\Theta(n^3)}$

Alg 5 $\Theta(n \log n)$

these are not going to scale well with n

Which alg do we want to use?

For alg 2 + 5 we can do more careful analysis to find constant C n log n

For both & there's a reasonable (~ >10%) difference then we can pick the one with smaller constant

In the end you may choose to implement & test algs 2 & 5.

Comparison between standard mathematical functions

logarithmic linear quadratic cubic

$\log_2 n$	$\sqrt{n}$	n	$n \log_2 n$	n^2	n^3
0	1.0	1	0	1	1
1	1.4	2	2	4	8
2	2.0	4	8	16	64
3	2.8	8	24	64	512
4	4.0	16	64	256	4,096
5	5.7	32	160	1,024	32,768
6	8.0	64	384	4,096	262,144
7	11.3	128	896	16,384	2,097,152
8	16.0	256	2,048	65,536	16,777,216
9	22.6	512	4,608	262,144	134,217,728
10	32.0	1,024	10,240	1,048,576	1,073,741,824
11	45.3	2,048	22,528	4,194,304	8,589,934,592
12	64.0	4,096	49,152	16,777,216	68,719,476,736

Divide-and-Conquer Algorithms

Note Title 9/3/2007

1. <u>Divide</u> - divide problem into subproblems (smaller input for same problem)

Conquer

2. Recursively solve subproblems (or directly solve when input reaches a termination condition)
3. <u>Combine</u> - use subproblem sols to solve given problem