Analysis

expected

(What is height (# of levels in tallest tower)?

X (How many times per level do we expect to move (or peak) forward)?

Let n be # elements (towers)

<u>Claim</u> expect $n \cdot p^i$ elements
in level i list

For $p = 1/4$

$i=0$	n	n
$i=1$	$n \cdot p$	$n/4$
$i=2$	$n \cdot p^2$	$n/16$
	$\vdots$	$\vdots$

Can prove by induction.

When is $n \cdot p^i = 1/p$

when $p = 1/4$
only 4
items
expected
at level i

Solve for i

$n = \left(\frac{1}{p}\right)^{i+1} \Rightarrow i+1 = \log_{1/p} n$

tallest
tower

$i = \log_{1/p} n - 1$

Level when
expect $1/p$
elements to
remain

$E[\text{height}] =$
$\log_{1/p} n + \frac{1}{1-p}$

Ex $p = 1/4$

$E[\text{height}] = \log_4 n + 4/3$

We'll finish analysis next time.

Skip List

Overview

	As a function of p	$p = 1/2$	$p = 1/e$	$p = 1/4$
space usage	$2n/(1-p) + 4\log_{1/p} n$	$4n + 4\log_2 n$	$\approx 3.2n + 2.8\log_2 n$	$\approx 2.67n + 2\log_2 n$
search cost	$(\log_{1/p} n)/p$	$2\log_2 n$	$\approx 1.88\log_2 n$	$2\log_2 n$

Ordered Collection Overview

Note Title 11/2/2007

Skip List Analysis

Analysis

What is expected height (# of levels in tallest tower)?

X How many times per level do we expect to move (or peak) forward)?

Last time argued that expect
$n \cdot p^i$ elements in level i list.

Find i for which $n \cdot p^i = 1/p$

For $p = 1/4$, level when you only expect 4 elements to remain

$$n = \left(\frac{1}{p}\right)^{i+1}$$

roughly $\log_{1/p} n$ levels

$$\log_{1/p} n = i + 1$$

$$\boxed{i = \log_{1/p} n - 1}$$

So at level $\log_{1/p} n - 1$ expect only $1/p$ elements to remain

\# levels until this point $\log_{1/p} n$

$0, 1, 2, \ldots, \log_{1/p} n - 1$

Can prove

$$E\left[\begin{array}{c}\text{height of}\\ \text{tallest tower}\end{array}\right] = \log_{1/p} n + \frac{1}{1-p}$$

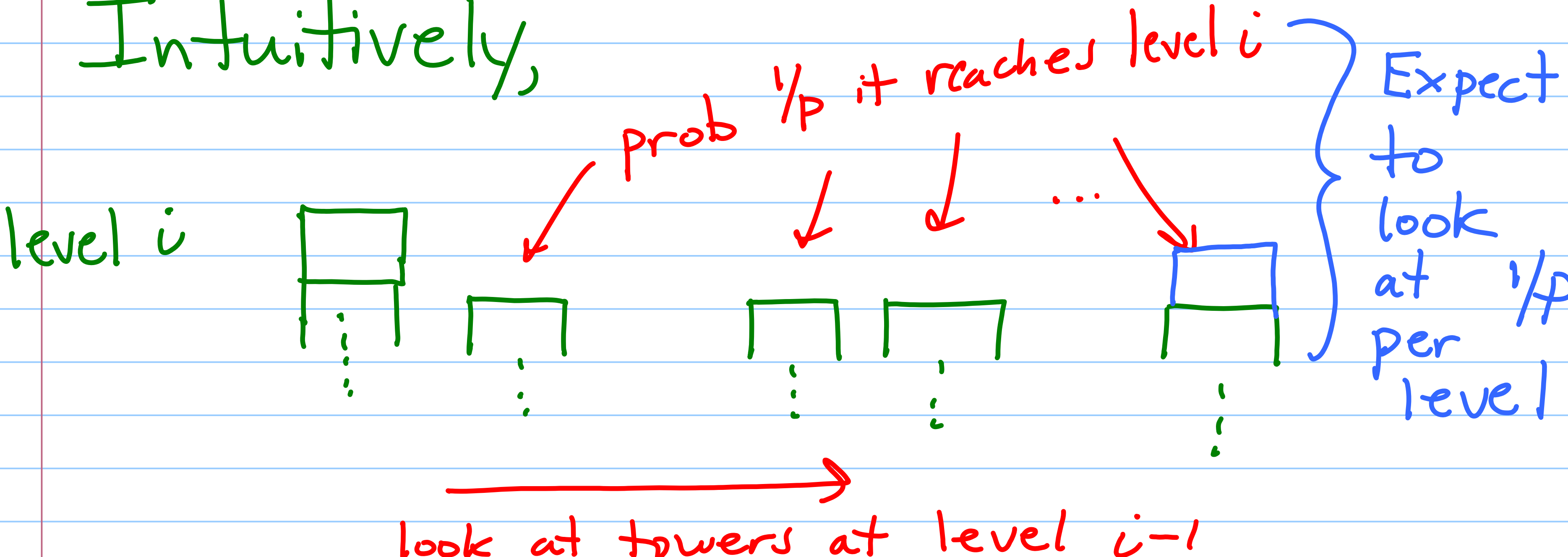
$p = 1/4$

$$\log_4 n + \frac{4}{3}$$

Search Cost

Page 615 goes over more formally.

Intuitively,



So expect $\log_{1/p} n + \frac{1}{1-p}$ levels ← $p = 1/4$, $\log_4 n + \frac{4}{3}$

$$\log_4 n = \frac{\log_2 n}{\log_2 4} = \frac{\log_2 n}{2}$$

+ expect to consider $1/p$ elements per level

$$\Rightarrow E[\text{search time}] \approx \frac{1}{p} \log_{1/p} n = \frac{1/p \log_2 n}{\log_2 1/p}$$

For $p = 1/4$, $E[\text{search time}] \approx 2 \log_2 n$

Space Usage

What is expected height of a tower

$$E[h] = \sum_{i=0}^{\infty} i\, p^{i-1}(1-p) = \frac{1}{1-p}$$

i-1 times continue (occurs with prob p)

last time stop (occurs prob 1-p)

next/prev refs (dominates space usage)

$$\approx \underbrace{n}_{\text{\# towers}} \cdot \underbrace{\frac{1}{1-p}}_{\text{expected height}} \cdot \underbrace{2}_{\text{next + prev per level of each tower}} + \underbrace{4\left(\log_{1/p} n + \frac{1}{1-p}\right)}_{\text{for head + tail}}$$

$$= \frac{2n}{1-p} + 4\log_{1/p} n + \frac{4}{1-p}$$

next/prev refs in skiplist for $p = 1/4$

$$2\tfrac{2}{3}n + 2\log_2 n + \frac{8}{3} \approx 2\tfrac{2}{3}n$$

left, right, parent refs in red-black tree

$$3n$$

parent, child, next, prev refs in B^+-tree with $t=2$

parent, next, prev

On average $\sim n/2$ leaves $+ \sim \frac{3n}{4}$ internal nodes ← parent + 4 child refs

$$\Rightarrow \sim 3(n/2) + 5(3n/4) = \frac{21n}{4} = 5\tfrac{1}{4}n$$

Skip List overview

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value of p to
minimize search
cost