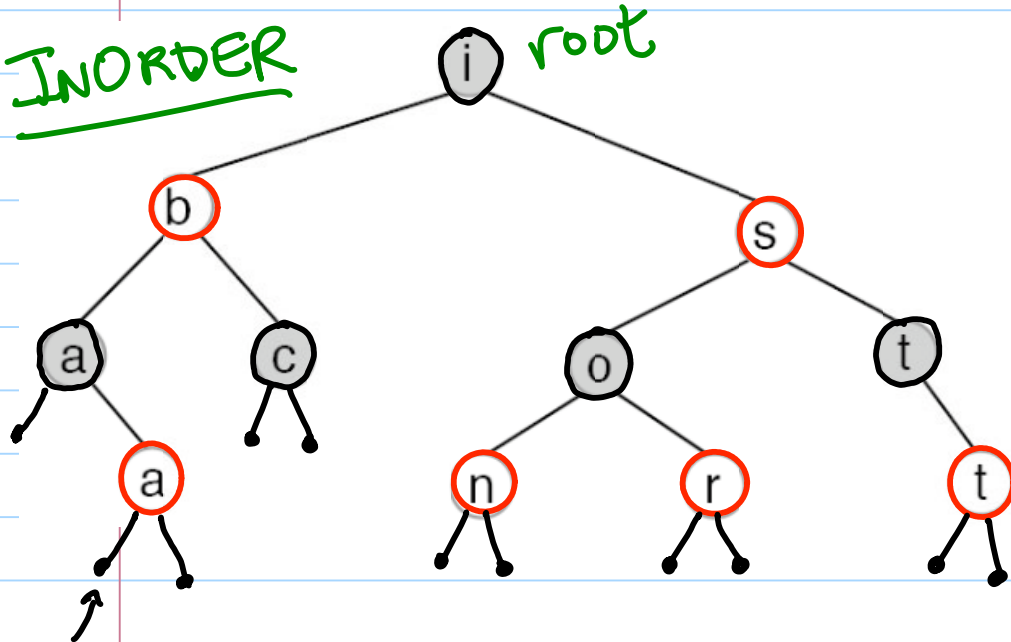


Red-Black Trees (Balanced Binary Search Trees)

Note Title

10/22/2007

INORDER



Frontier
(instead
of null)

black height of 2

Representation
Properties

Black Balanced - the # of black nodes on any path from root to a leaf is the same (black height)

No Double Reds -
No red node has a red child

Root Black -
root is black

max # nodes on a path
from root to a leaf

Upperbound on Height

Show height $\leq 2 \log_2(n+1)$

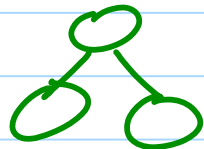
A complete binary tree of l levels
has $2^l - 1$ nodes

$l=1$



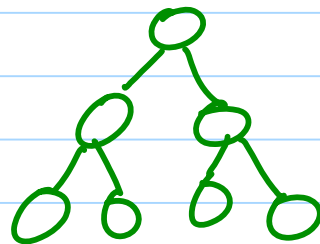
$$2^1 - 1 = 1$$

$l=2$



$$2^2 - 1 = 3$$

$l=3$



$$2^3 - 1 = 7$$

prove
above
claim by
induction

In a red-black tree with black height bh

nodes $\rightarrow n \geq \underbrace{2^{bh} - 1}_{\text{\# black nodes}}$

By algebra $bh \leq \log_2(n+1)$

By No Double Reds & Root Black at least half of nodes on any path from root to a leaf are black $\Rightarrow \boxed{h \leq 2 \log_2(n+1)} = O(\log n)$