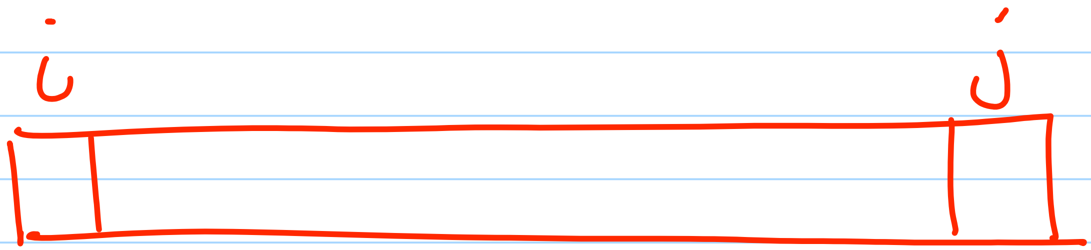
Overview of Analysis

- Dominant cost is comparison performed by partition
- Only aspect that affects time complexity is # elements in subarrays for recursive calls (at each level of recursion)



j-i+1 elements

$\leq \frac{1}{j-i+1}$ prob i+j get compared

Adversary Lower Bound Technique

Note Title 9/25/2007

Let's first complete our discussion of quicksort.

Expected # of comparisons

$$\underbrace{\sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1}}_{\text{over all } i,j \text{ pairs}} \text{prob}\left(\text{elements in positions } i \,\&\, j \text{ are compared}\right)$$

What is probability that the elements in positions i + j are compared

sort this subarray and it contains pos i + pos j

this occurs if u_i or u_j are selected as pivot

$$\text{Prob}\left[u_i + u_j \text{ compared}\right] = \frac{2}{\#\text{ elements in subarray}} \leq \frac{2}{j-i+1}$$

min size $j-i+1$

So,

Expected # of comparisons

$$\sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} \text{prob}\left(\begin{array}{c}\text{elements in positions} \\ i \,\&\, j \text{ are compared}\end{array}\right)$$

$$\leq \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} \frac{1}{j-i+1} = \sum_{i=0}^{n-2} 2\left(\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-i}\right)$$

$$\leq \sum_{i=0}^{n-2} 2\left(1 + \frac{1}{2} + \cdots + \frac{1}{n}\right)$$

$$\leq \sum_{i=0}^{n-2} 2(\ln n + 1) = 2(n-1)(\ln n + 1)$$

$$= \Theta(n \log n)$$