

Correctness of Dijkstra's Alg

ASIDE

Greedy Algs aren't always optimal

Knapsack that can hold 100 lbs

80, 50, 40

Greedy alg: Take highest value that fits & repeat

Dijkstra's Alg Proof of Correctness

Prove following invariants shortest path tree

1. For vertex $u \in T$, $u.dist$ is length of shortest path from s to u

2. For vertex $u \in Q$, tag for u is length of shortest path from s to u with intermediate vertices restricted to T Priority Queue

Inductive Proof

Base $T = \emptyset$, $Q = \{s\}$

rest is in U
undiscovered
vertices

Prop 1 holds vacuously

Prop 2 holds since tag for s is 0

Inductive Step

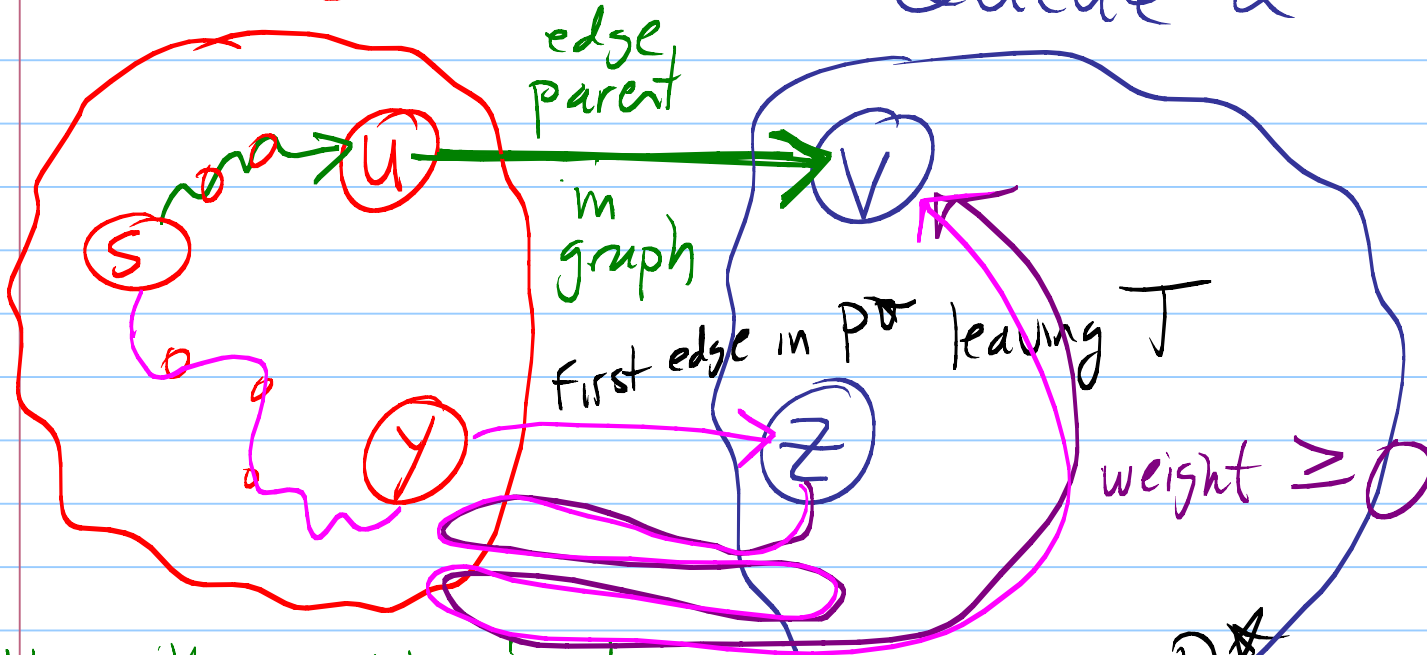
Suppose it fails at some point.

Proof by contradiction.

Tree T

Priority Queue Q

$$tag_v \leq tag_z$$



weight P^*
at least as
large as
weight of
green path.

path with weight v 's tag

Better path from S to V

Extract Max gives V + this causes first failure of one of the properties