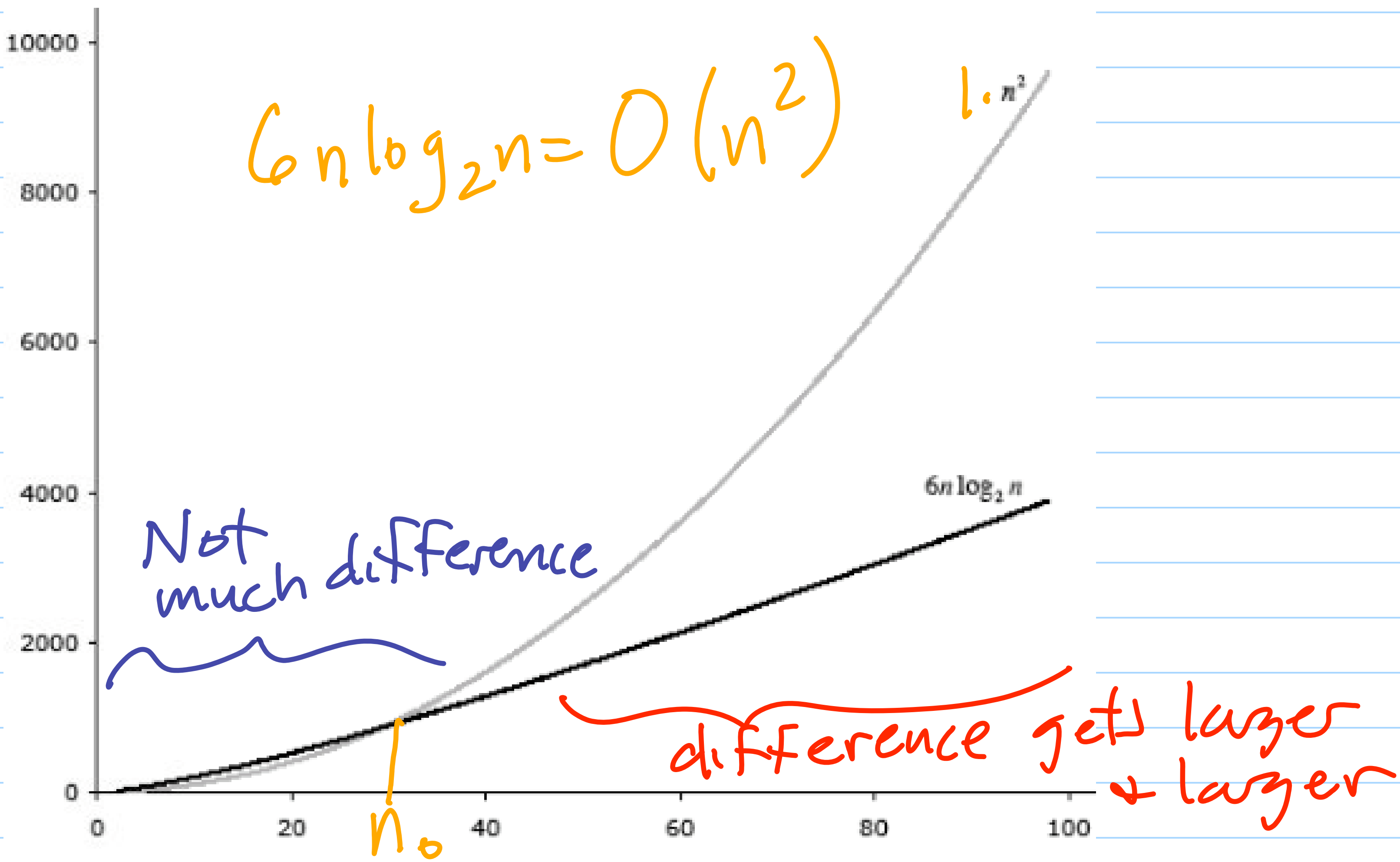
# Asymptotic Notation

Note Title 9/11/2007



While $n$ doesn't grow arbitrarily large, it's useful to consider when it does

$$\lim_{n \to \infty} \frac{6n\log_2 n}{n^2} = 0$$

$$\lim_{n \to \infty} \frac{n^2}{6n\log_2 n} = \infty$$

In general

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \begin{cases} 0 & f(n) \text{ is asymptotically slower growing than } g(n) \\ \text{constant} & f(n) \;\&\; g(n) \text{ grow at same asymptotic rate} \\ \infty & f(n) \text{ is asymptotically faster growing than } g(n) \end{cases}$$

think of as time complexities

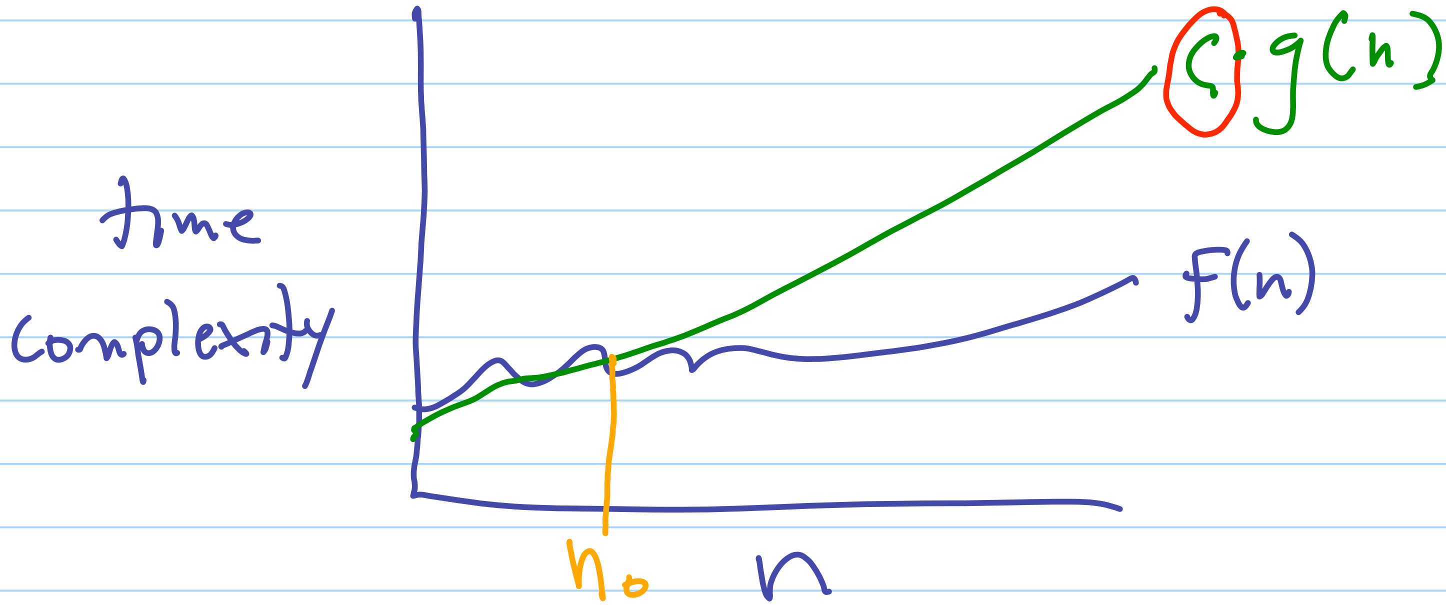
assume $f(n) \geq 0$

$g(n) \geq 0$

# Big-Oh Notation    asymptotic upper bound $\leq$

$f(n) = O(g(n))$ if $\exists$ constants $c$ & $n_0$

such that $\forall n \geq n_0$ $f(n) \leq c \cdot g(n)$



exs $\frac{3}{2}n^2 + \frac{7n}{2} - 4 = O(n^2)$ let $c=2$, $n_0 = 6$ or $c=3$, $n_0=1$

this is really a short hand for set containment ( $\in$ )

$O(n^2)$ is the set of functions

$\{ f(n) \mid f(n) \text{ is } O(n^2) \}$

such that

$\forall n \geq 6 \quad \frac{3}{2}n^2 + \frac{7n}{2} - 4 \leq 2n^2$

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

More exs

asymp. "$\leq$"

$$6n\log_2 n = O(n^3)$$

Is it meaningful to say

$$T(n) \geq O(n^2) \text{ ?}$$

lower bound

notation for an upper bound

$$1 = O(n^2)$$

all we can conclude is $T(n) \geq 1$

Is it meaningful to say
$T(n) \leq O(n^2)$ ? yes

$T(n)$ asymptotically grows no faster than $c \cdot n^2$

The same as $T(n) = O(n^2)$

Is $6n^3 = O(n^2)$ ? No.

When $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$ or constant

then $f(n) = O(g(n))$

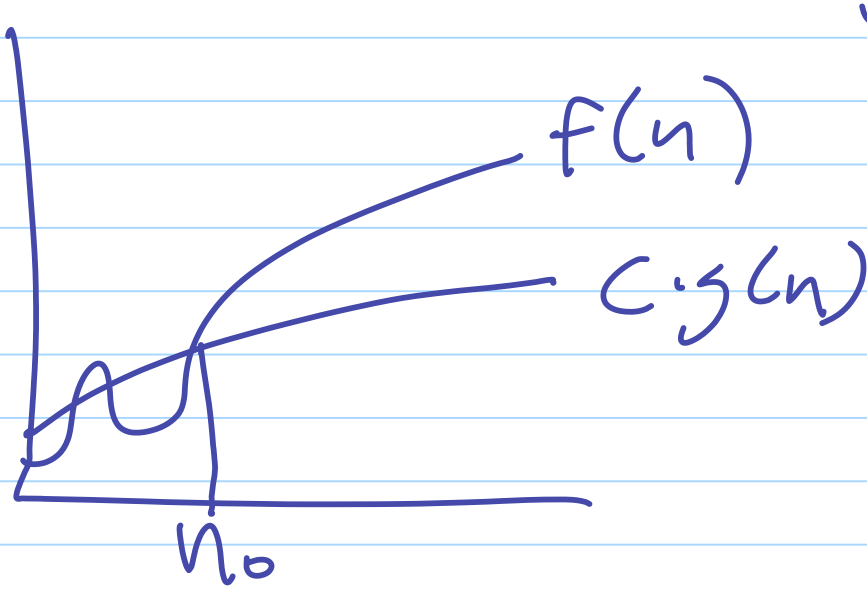
When $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$ then

$f(n) \neq O(g(n))$

$\Omega$ $\geq$

# Big Omega Notation (asymptotic lower bound)

$f(n) = \Omega(g(n))$, if $\exists$ constants $c$ & $n_0$ such that $f(n) \geq c \cdot g(n)$ $\forall n \geq n_0$



$f(n) = O(g(n))$

time complexity

$\underbrace{g(n)}$ simple mathematical function

$n,\ n^2,\ n\log n,\ \sqrt{n}$

Time complexity to find closest pair of points among $n$ points $= \Omega(n \log_2 n)$

$f(n) =_{\lim} g(n)$

equiv $f(n) \geq_{\lim} g(n)$

$\Theta$ $f(n) \leq_{\lim} g(n)$

Big-Theta

$f(n) = \Theta(g(n))$ is defined as

$f(n) = O(g(n))$ & $f(n) = \Omega(g(n))$

$\leq_{\lim}$ } asymptotic bound (within a constant)

$f(n) \leq_{\lim} g(n)$ $\quad f(n) = O(g(n))$

$f(n) <_{\lim} g(n)$ $\quad f(n) = o(g(n))$

$f(n) =_{\lim} g(n)$ $\quad f(n) = \Theta(g(n))$

$f(n) \geq_{\lim} g(n)$ $\quad f(n) = \Omega(g(n))$

$f(n) >_{\lim} g(n)$ $\quad f(n) = \omega(g(n))$

little omega

think

$f(n) = O(g(n))$ $\quad f(n) \leq_{\lim} g(n)$

$+ \quad g(n) = O(h(n))$ $\quad g(n) \leq_{\lim} h(n)$

$f(n) = O(h(n))$ $\quad f(n) \leq_{\lim} h(n)$

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$f(n) = O(g(n))$ $\quad f(n) \leq_{\lim} g(n)$

$g(n) = \Omega(f(n))$ $\quad g(n) \geq_{\lim} f(n)$

# Overview

| Asymptotic Notation | $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ | Alternate Notation |
|---|---|---|
| $f(n) = O(g(n))$ ("big-Oh") | constant or 0 | $f(n) \leq_{lim} g(n)$ |
| $f(n) = \Omega(g(n))$ ("big-Omega") | constant or $\infty$ | $f(n) \geq_{lim} g(n)$ |
| $f(n) = \Theta(g(n))$ ("big-Theta") | constant | $f(n) =_{lim} g(n)$ |
| $f(n) = o(g(n))$ ("little-oh") | 0 | $f(n) <_{lim} g(n)$ |
| $f(n) = \omega(g(n))$ ("little-omega") | $\infty$ | $f(n) \geq_{lim} g(n)$ |