Analyzing Divide-and-Conquer Algs

Note Title 9/16/2007

Reminder

$$T(n) = aT(n/b) + f(n)$$

- $T(n)$: # statements (time) when input is size n
- a: # of subproblems that we recursively solved
- n/b: size of each subproblem (really, $\lfloor n/b \rfloor$ or $\lceil \frac{n}{b} \rceil$)
- $f(n)$: total time for divide and combine steps together

Base: $T(c) = \Theta(1)$ — problem size where you no longer recurse

Merge sort

split is $\Theta(1)$
merge is $\Theta(n)$

$a=2,\ b=2,\ f(n)=\Theta(n)$

$T(n)=2T(n/2)+\Theta(n)$

Divide-and-conquer closest pair

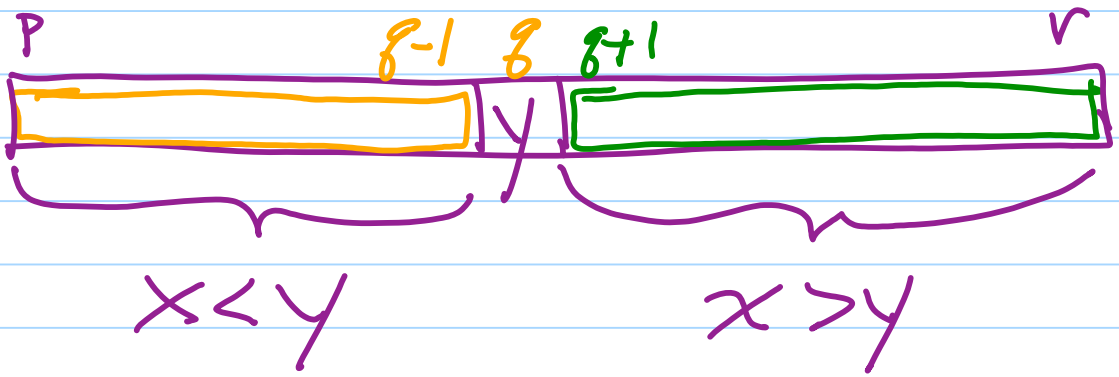
$a=2,\ b=2,\ f(n)=\Theta(n)$

both divide & combine steps take linear time

Binary Search

Problem: Given a sorted array and a value x

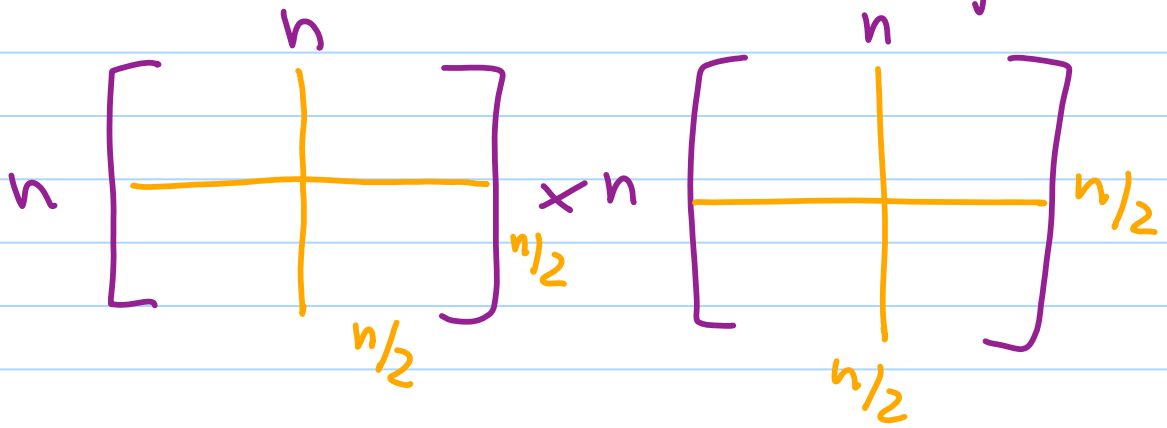
Asked if x is in the array



$T(n) = T(n/2) + \Theta(1)$

$a = 1, \quad b = 2$

Strassen Matrix Multiplication



There are 7 multiplications of $n/2$ by $n/2$ matrices which can be combined in $\Theta(n^2)$ time to get the desired product

$$T(n) = 7T(n/2) + \Theta(n^2)$$

$a = 7,\ b = 2$