Return to problem of sorting \( n \) comparable elements with a comparison based alg.

As adversary, what should I put in list \( L \)?

\[
\# \text{ comparisons} \geq \lceil \log_2 n! \rceil
\]
Ex) $n = 3$

\[ a_0, a_1, a_2 \]

\[ \begin{array}{c|ccc}
\text{no} & 1, 2, 3 \\
\text{no} & 2, 3, 1 \\
\text{yes} & 3, 1, 2 \\
\text{yes} & 2, 1, 3 \\
\text{no} & 1, 3, 2 \\
\end{array} \]

From alg $a_0 \geq a_1$, ?

Yes

There are all different permutations

What alg will output is something like $a_1 < a_2 < a_0$

Can't have both $[3, 2, \_]$
In general L begins with $N!$ inputs (one for each permutation).

Adv strategy: Answer each comparison based on majority in $L$. 
Stirling's approximation

\[ n! \geq \left( \frac{n}{e} \right)^n \]

# comp \[\geq \left\lfloor \log_2 \left( \frac{n}{e} \right)^n \right\rfloor\]

\[ = \left\lfloor n \left( \log_2 n - \log_2 e \right) \right\rfloor \]

\[ = \left\lfloor n \log_2 n - n \log_2 e \right\rfloor = \Theta(n \log n) \]