Overview of Analysis

- Dominant cost is comparison performed by partition

- Only aspect that affects time complexity is \# elements in subarrays for recursive calls (at each level of recursion)
Adversary Lower Bound Technique

Let's first complete our discussion of quicksort.

\[
\text{Expected \# of comparisons} = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} \text{prob (elements in positions } i, j \text{ are compared)}
\]

over all \(i, j\) pairs
What is probability that the elements in positions $i$ and $j$ are compared

Sort this subarray and it contains pos $i$ and pos $j$

This occurs if $U_i$ or $U_j$ are selected as pivot

$$\text{Prob}[U_i + U_j \text{ compared}] = \frac{2}{\# \text{elements in subarray}} \leq \frac{2}{j - i + 1}$$

Min size $\rightarrow j - i + 1$
So,

Expected # of comparisons

\[
\sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} \text{prob (elements in positions } i \text{ and } j \text{ are compared)}
\]

\[
\leq \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} \frac{1}{j-i+1} = \sum_{i=0}^{n-2} 2 \left( \frac{1}{2 + \frac{1}{3} + \cdots + \frac{1}{n-i}} \right)
\]

\[
\leq \sum_{i=0}^{n-2} 2 \left( 1 + \frac{1}{2} + \cdots + \frac{1}{n} \right)
\]

\[
\leq \sum_{i=0}^{n-2} 2 (\ln n + 1) = 2 (n-1) (\ln n + 1) = \Theta (n \log n)
\]