Correctness of Dijkstra's Alg

Aside

Greedy Algs aren't always optimal

Knapsack that can hold 100 lbs

80, 50, 40

Greedy alg: Take highest value that fits & repeat
Dijkstra’s Alg Proof of Correctness

Prove following invariants

1. For vertex \( u \in T \), \( u.d \) is length of shortest path from \( S \) to \( u \)

2. For vertex \( u \in Q \), tag for \( u \) is length of shortest path from \( S \) to \( u \) with intermediate vertices restricted to \( T \)
Inductive Proof

Base \( T = \emptyset \), \( Q = \{ S \} \) rest is in \( U \) undiscussed vertices

Prop 1 holds vacuously
Prop 2 holds since tag for \( S \) is 0

Inductive Step

Suppose it fails at some point.
Proof by contradiction.
Tree $T$

Priority Queue $Q$

Path with weight $w$ to $v$ with tag $\tau$:

- First edge in $P^*$ leaving $T$ with weight $\geq 0$
- Path with weight $w$ to $v$ with tag $\tau$

Better path from $s$ to $v$:

Extract Max gives $v$ and this causes first failure of one of the properties.

$\tau v \leq \frac{\tau}{2}$

Weight $p^*$ at least as large as weight of green path.